# Fatigue Strength Assessment of Spot-Welded Lap Joint Using Strain Energy Density Factor

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One of the recent issues in design of the spot-welded structure such as the automobile body is to develop an economical prediction method of the fatigue design criterion without additional fatigue test. In this paper, as one of basic investigation for developing such methods, fracture mechanical approach was investigated. First, the Mode I, Mode II and Mode III, stress intensity factors were analyzed. Second, strain energy density factor (S) synthetically including them was calculated. And finally, in order to decide the systematic fatigue design criterion by using this strain energy density factor, fatigue data of the $\Delta P-N_f$ obtained on the various in-plane bending type spot-welded lap joints were systematically re-arranged in the $\Delta S$-$N_f$ relation. And its utility and reliability were verified by the theory of Weibull probability distribution function. The reliability of the proposed fatigue life prediction value at $10^7$ cycles by the strain energy density factor was estimated by 85%. Therefore, it is possible to decide the fatigue design criterion of spot-welded lap joint instead of the $\Delta P-N_f$ relation.

**Key Words** : Spot Welding, IB (in-plane bending) Type Specimen, Fatigue Strength, Stress Intensity Factor, Strain Energy Density Factor, Reliability

## 1. Introduction

As fatigue strength of the spot-welded joint affects structural rigidity and durability of the spot-welded structures, safety and structural integrity of the structure are generally decided by fatigue strength of the spot weld. Therefore, accurate stress analysis and systematical fatigue strength assessment of spot-welded joint are very important to determine its long-life fatigue design criterion. However, it is very difficult to determine the fatigue design criterion with the actual structure, directly. Thus, after estimating fatigue strength with simulated specimen satisfying the structural characteristic of the actual structure, the fatigue design criterion is generally determined and applied to the design of the spot-welded structure.

In general, the most typical and traditional method to determine the fatigue design criterion is $\Delta P-N_f$ curve. However, it is difficult to determine systematical fatigue design criterion considering the effects of the various geometrical factors of spot-welded joints, because the relationship between fatigue load range ($\Delta P$) and fatigue life ($N_f$) is varied according to the material properties, the geometrical factors, and the mechanical condition. Thus, many researchers have numerically and experimentally investigated to develop the method of systematic fatigue strength assessment of various spot-welded joints (Hujimoto, 1979; JSAE Committee, 1987; Bae et al., 1988; Bae, 1991, Oh, 1998, Sohn, 1998). Among them, the fracture mechanical approaches

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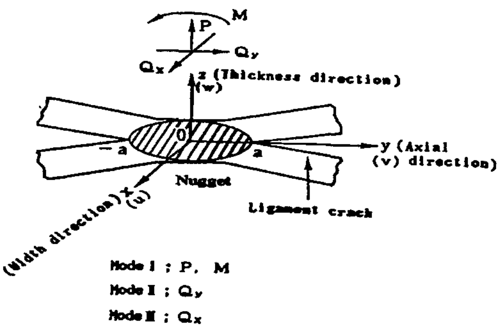



**Fig. 1** Mechanical model of IB type spot-welded lap joint

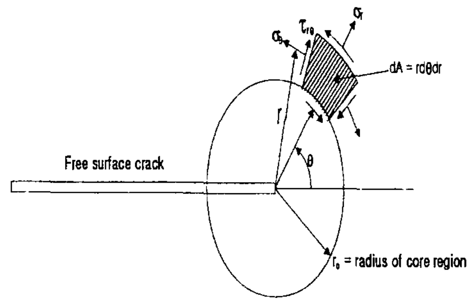



**Fig. 2** 2-dimensional core region surrounding the crack tip

can be also found. But, almost all the fracture mechanical approaches have treated tensile shear type joint, T-type joint, cross tension type joint which are considered as single mode or mixed mode including just Mode I and Mode II. It is difficult to find out the cases simultaneously considering Mode I, II, and III like IB type joint.

In this paper, as one of the basic investigation for developing the methods satisfying such requirements, fracture mechanical approach was investigated by considering around the nugget edge of in-plane-bending type, which is defined as IB type, spot-welded lap joint as a ligament crack shown in Fig. 1. The stress intensity factors were analyzed on the Mode I, II and III, respectively, and then, the strain energy intensity factor (S) synthetically including the fracture mechanical modes was calculated. $\Delta P - N_f$ obtained on the various spot-welded joints were systematically rearranged in a $\Delta S - N_f$ relation by using strain energy density factor (S). And its utility and reliability were verified by the theory of Weibull probability distribution (Patrick and O'connor, 1995).

## 2. Numerical Analysis of the Strain Energy Density Factor Around the Spot Weld

### 2.1 The theory of the strain energy density factor

G. C. Sih proposed the strain energy density factor (S) Sih (1973, 1974). This theory is based on the following assumptions that were made in the strain energy density theory: 1) Crack growth is directed along the line from the center of the spherical core (Fig. 2) to the point on the spherical surface with the maximum strain energy density factor ($S_{min}$). 2) growth along this direction begins when $S_{min}$ reaches the maximum critical value $S_c$ which the material will tolerate. The total strain energy per unit volume $(dW/dV)$ stored in the element $(dV = dx \cdot dy \cdot dz)$ of near the ligament crack tip is as follows:

$$\frac{dW}{dV} = \left[\frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{V}{E}(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)\right. \\ \left. + \frac{1}{2\mu}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right] \quad (1)$$

Substituting the local stresses (Sih, 1975) into Eq. (1):

$$\frac{dW}{dV} = \frac{1}{r}(a_{11} \cdot K_I^2 + 2a_{12} \cdot K_I \cdot K_{II} + a_{22} \cdot K_{II}^2 \\ + a_{33} \cdot K_{III}^2) \quad (2)$$

Equation (2) has a singularity of $\frac{1}{r}$ at the crack tip. Using the relation $dW/dV = S/r$, the strain energy density factor is defined as:

$$S = a_{11}K_1^2 + 2a_{12}K_1K_2 + a_{22}K_2^2 + a_{33}K_3^2 \quad (3)$$

Where the coefficients $a_{ij}$(I, j=1, 2, 3) in Eq. (3) are given by:

$$a_{11} = \frac{1}{16G}(1 + \cos\theta)(\chi - \cos\theta),$$

$$a_{12} = \frac{1}{16G}\sin\theta\{2\cos\theta - (\chi - 1)\}$$

$$a_{22} = \frac{1}{16G}\{(\chi + 1)(1 - \cos\theta) + (1 + \cos\theta) \\ (3\cos\theta - 1)\}, \quad a_{33} = \frac{1}{4G} \quad (4)$$

Where $\chi=\frac{(3-\nu)}{(1+\nu)}$ : plane stress condition, and $\chi=3-4\nu$ : plane strain condition.

G = shear modulus

And, the stress intensity factor on Mode I, II and III at near the crack tip is given by:

$$K_I=\frac{2G\cdot(2\nu)\cdot\sqrt{\frac{2\pi}{r}}}{\sin\frac{\theta}{2}\cdot\left(x+1-2\cos^2\frac{\theta}{2}\right)},$$

$$K_{II}=\frac{2G\cdot(2u)\cdot\sqrt{\frac{2\pi}{r}}}{\sin\frac{\theta}{2}\cdot\left(x+1+2\cos^2\frac{\theta}{2}\right)},$$

$$K_{III}=\frac{2G\cdot(2w)\cdot\sqrt{\frac{2\pi}{r}}}{2\cdot\sin\frac{\theta}{2}} \tag{5}$$

### 2.2 Numerical analysis of the strain energy density factor

When tensile shear load is applied to the IB type single spot-welded lap joint, in-plane shear force (P), in-plane bending force (Q), and out of plane bending moment (M) act on the spot-weld. Thus, a very complicate deformation occurs by these forces as shown in Fig. 1. By this deformation mechanism, fatigue cracks are generally initiated from the nugget edge on the inner surface of the plate by stress concentration, and propagate to the outer surface of the plate. (Sohn, 1998). Therefore, it is very important to calculate the accurate stress distribution and deformation conditions around the nugget edge for reasonable fatigue strength assessment.

In fracture mechanical approach, as spot-welded joint commonly shows large-scale deformation characteristics like Fig. 1. In this case, calculating the stress intensity factor for each mode may be more reasonable by displacement approach considering its deformation characteristics than by stress approach. Thus, the stress intensity factor for each mode of IB type spot-welded joint was calculated by using the displacement approach. Figure 3 shows the configuration of IB type spot-welded lap joint. And, Fig. 4 is a 3-D FEA model to calculate stress distribution and deformation on IB type spot-welded lap joint. This joint model is to simulate the spot-welded bus window pillar joint sustaining in-plane force by warping of the body structure.

The material used is a cold rolled high strength steel sheet for automobile body that is named by SPCC, and the same material properties were used for FEA. Its chemical composition and mechanical properties at room temperature are illustrated in Tables 1 and 2, respectively.

Dimensions of the reference model are as follows; plate thickness (t):1 mm, plate width (w):30 mm, lapped length (2L):30 mm and joint angle ($\theta$):2.5°. 3D solid brick elements were entirely

**Table 1** Chemical composition of specimen(Wt, %)

| | C | Si | Mn | P | S | Ni | Al | Fe |
|---|---|---|---|---|---|---|---|---|
| SPCC | 0.12 | 0.01 | 0.127 | 0.015 | 0.007 | 0.025 | 0.045 | Rem |

**Table 2** Mechanical properties of specimen

| | Tensile Strength (MPa) | Yield Strength (MPa) | Elongation (%) |
|---|---|---|---|
| SPCC | 307.0 | 168.4 | 47 |

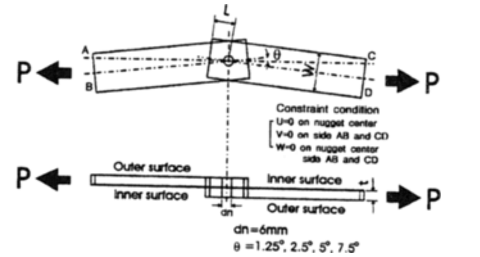



**Fig. 3** Configuration of IB type spot-welded lap joint

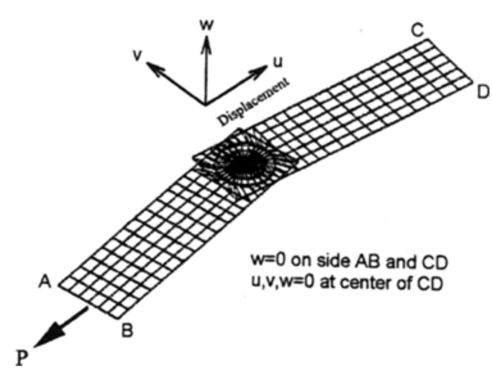



**Fig. 4** The 3-dimensional finite element analysis model of IB type spot-welded lap joint

used. Mesh generation for the upper and lower plate was symmetrically performed. The total number of elements and nodes used is about 1164 and 1992, respectively. The upper and lower plate had one layer solid brick element. The weld nugget area was fine meshed and nugget size was modeled to the same size of electrode that was used in actual industrial field. Particularly, the weld nugget was formed by getting together the node numbers of the elements contacting each other on the inner surfaces of the upper and lower plate. Boundary and load conditions were applied as same as the fatigue test condition.

For confirming deformation conditions, tensile shear load of 9.81 MPa was previously applied to the reference model in direction illustrated in Fig. 4. But, when systematically estimated fatigue strength of the joints, the strain energy density factor was calculated with actual loads applied to each fatigue specimen. MSC/NASTRAN, a commercial FEA package, was used for the numerical analysis.

### 2.3 Numerical analysis results on the strain energy density factor

$K_I$, $K_{II}$ and $K_{III}$ calculated by FEA are shown in Fig. 5 at same load condition. According to the joint angle ($\theta$) increases, $K_I$ shows nearly constant values, but $K_{II}$ is decreased, while $K_{III}$ is monotonically increased. In particular, in the case of $\theta=0°$, that is named as TS (tensile shear) type spot-welded lap joint, it is $K_{II}>K_I$, and $K_{III}=0$. It means that the most influential parameter in this case is $K_{II}$. Thus, in fracture mechanical analysis for TS type single spot-welded lap joint,

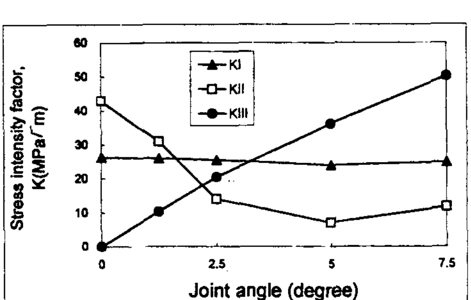



**Fig. 5** Variation of stress intensity factors against joint angle. (P=9.81MPa)

it is generally handled as the mixed mode considering just $K_I$ and $K_{II}$ (JSAE Committee, 1987). But, when the joint angle ($\theta$) becomes unequal to zero, it is found that IB (in-plane bending) type spot-welded joint must be considered as being in the state of the mixed mode of $K_I$, $K_{II}$ and $K_{III}$. And, in range of the joint angle $(\theta)\geq3.0$, the most influential fracture mechanical parameter becomes $K_{III}$. Therefore, considering these characteristics, the strain energy density factor synthetically including $K_I$, $K_{II}$, and $K_{III}$ was calculated from the results of Fig. 5.

As mentioned above, when tensile shear load is applied to IB type spot-welded joint, three kinds of the load components partially act on the spot weld. These load components cause very complicate deformations, and affect the stress intensity factors as well as the strain energy density factor. By this mechanism, the strain energy density factor is also influenced by the geometrical factors such as the joint angle, plate thickness, and width of the plate of IB type spot welded lap joint (Oh, 1998). It was known that the strain energy density factor was decreased as the plate thickness and width of the plate increased with the joint angle. In particular, because the susceptibility of the joint angle and the plate thickness on the strain energy density factor are very large, it is necessary to consider their effects on the fatigue design of the spot- welded structures such as the bus window pillar member which must consider static and fatigue strength by in-plane force (Kim, 1999).

In this study, in order to determine the systematic fatigue design criterion for IB type spot-welded joints having various dimensions and subjected to tensile shear fatigue load, the $\Delta P-N_f$ relation was systematically rearranged by the strain energy density factor in around the ligament crack of the spot weld.

## 3. Load-Life ($\Delta P-N_f$) Relation of IB Type Spot-Welded Joint

### 3.1 Specimen and test method

The fatigue specimens were prepared with the same welding parameters, configurations and

**Table 3** Welding condition of specimen

| | Electrode force (F) | Welding Current (I) | Welding Time (Cycle) |
|---|---|---|---|
| Welding Condition | 1962 N | 8.3 kA | 15 Cycle |

**Table 4** Fatigue test condition

| Test condition | IB type specimen |
|---|---|
| Load Ratio ($R=P_{min}/P_{max}$) | 0 |
| Frequency (Hz) | 25 |
| Wave pattern | Sine |

material as the model for numerical analysis. The resistance spot welding was processed under the welding condition recommended from RWMA (1981) class-C illustrated in Table 3. The diameter of the electrode used for the spot welding is 6mm, which is normal size of electrode applying in actual field.

The fatigue test was conducted using a servo-hydraulic power system (MTS, 10ton). The fatigue tests for spot-welded joints such as TS (tensile shear) type specimen have been generally performed with the simple grips (Bae et al., 1988). By the way, in the case of IB type spot-welded joint subjected to tensile shear load, the simple grip cannot be used due to in-plane bending deformation generating in around the spot weld. Thus, in this test, in order to consider the effects of in-plane bending deformation and force by tensile shear load, the specially designed pin joint grip was used to control the in-plane bending effect.

The fatigue tests for spot-welded lap joint are generally conducted under the frequency of 21-130 Hz in air and room temperature, because the influence of the frequency on the characteristics of fatigue cracking is as small as negligible (Shigley and Mischke, 1989). The fatigue tests were therefore conducted under the condition illustrated in Table 4. The number of fatigue cycles to failure was decided as that of the cycles when fatigue crack came out on the outer surface of the specimen and its length was equal to nugget diameter. Fatigue limit of each specimen was decided as the load which crack was not initiated until cycles.

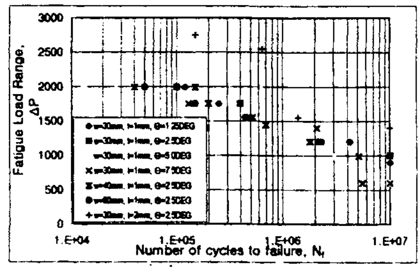



**Fig. 6** $\Delta P$-$N_f$ relation of spot-welded lap joints

### 3.2 Fatigue test results

Fatigue test results presented in the $\Delta P-N_f$ relation for various IB type spot-welded lap joints were shown in Fig. 6. Fatigue cracks were mostly generated in the near of the nugget edge and propagated to the out surface of the plate.

The influences of the geometrical factors on fatigue strength of the specimen were estimated. In high load and short life range, their influences except the plate thickness are not clearly estimated due to the complicate deformation characteristics of the thin plate. However, in the low load and long life range, their influences are certainly revealed. Under the same thickness condition, fatigue limit decreases as the joint angle of the specimen increases. This was due to that torque of the weld nugget by in-plane bending deformation linearly increased with the increase of the joint angle. However, the influence of the plate width on fatigue strength is not clearly identified.

$\Delta P-N_f$ data shown in Fig. 6 are scattered by the influences of the geometrical factors of the specimen. Although the influences of the geometrical factors on fatigue strength of the specimen could be found from these results, it was very difficult to determine the fatigue design criterion to systematically consider the influence of the geometrical factors on fatigue strength. Thus, it is necessary to determine the systematic and reasonable fatigue design criterion considering the influence of geometrical and mechanical factors.

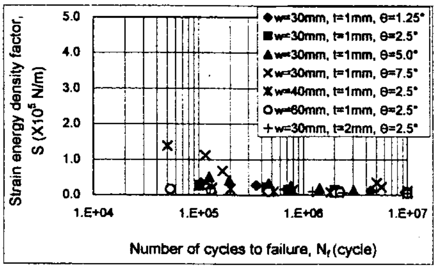
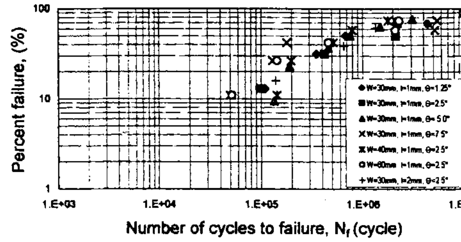



**Fig. 7** $S$-$N_f$ relation of spot-welded lap joints

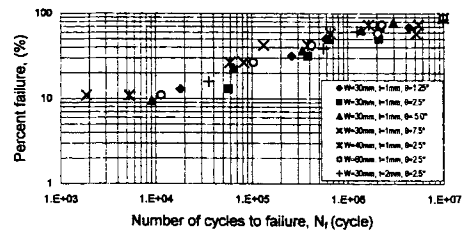
## 4. Systematical Fatigue Strength Assessment of IB Type Spot-Welded Joint

There are four basic approaches for fatigue analysis of welded components. They are the nominal stress approach, the structural hot spot stress or strain approach, the local notch stress or strain approach, and the fracture mechanical approach (Niemi, 1995). Many researchers have so far numerically and experimentally investigated on fracture mechanical fatigue analysis of spot-welded joint. For example, Pook (1975) evaluated fatigue strength of TS type spot-welded joint by using just $K_I$ and Linder et al. (1996, 1998) proposed the fatigue analysis result using equivalent stress intensity factor including $K_I$, $K_{II}$ and $K_{III}$. Fracture mechanical approaches for fatigue strength assessment of spot-welded joint have been so far investigated by using just $K_I$ or $K_I + K_{II}$ like them, but it is difficult to find the approaches including $K_I$, $K_{II}$ and $K_{III}$. Thus, in order to evaluate fatigue strength of IB type spot-welded joint that is need to consider as the mixed mode simultaneously including $K_I$, $K_{II}$ and $K_{III}$, the strain energy density factor (S) proposed by Sih (1975) was introduced.

Figure 7 shows systematical rearrangement of the $\Delta P - N_f$ relation by the $\Delta S - N_f$ relation. It is found that the $\Delta P - N_f$ data is more systematically and reasonably rearranged by the strain energy density factor at near the nugget edge of the spot weld. From Fig. 7, the fatigue limit of various IB type spot-welded lap joints subjected to tensile shear load was predicted in the range of $1.0 \sim 2.0 \times 10^3 N/m$.




(a) 2-parameter Weibull distribution function.




(b) 3-parameter Weibull distribution function

**Fig. 8** Comparison of Weibull distribution function

## 5. Reliability Verification of the Fatigue Strength Assessment Method Using the Strain Energy Density Factor

It has been well known that the theory of Weibull probability distribution is useful in statistical approach to fatigue strength and fatigue life prediction of weldment by the arc welding or the gas welding (Patrick and O'connor, 1995; Crowder et al., 1991; Kao, 1959). But, it seems to be not yet on the electric resistance spot welding. Thus, at first, applicability of Weibull probability distribution function to fatigue strength assessment of spot-welded lap joint was investigated. The model considered was IB type single spot-welded lap joint of Fig. 4.

Cumulative distribution functions by the 2- and 3-parameters Weibull distribution function on the fatigue data of spot-welded joint having various dimensions were compared in Fig. 8. It has been known that the Weibull distribution function presenting in log-log scale is desirable to be presented in a straight line for obtaining more accurate result from the transformation function and failure life relation (Patrick and O'connor, 1995). But the 2-parameters Weibull distribution

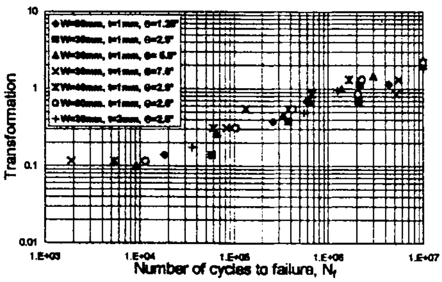



**Fig. 9** Relationship between transformation function and failure life

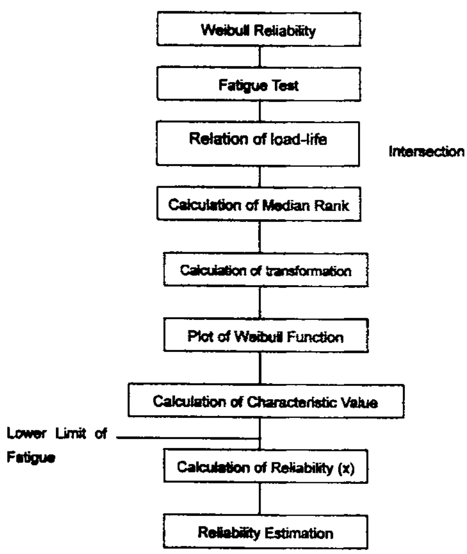



**Fig. 10** The procedure of reliability verification using 3-parameter Weibull distribution function

function did not satisfy the requirement in comparison with the 3-parameters. The relationship between transformation function calculated by the 3-parameters Weibull distribution function and the number of cycles to failure was plotted in Weibull probability paper of Fig. 9. These results also show good linearity.

Therefore, reliability estimation of the method to determine the fatigue design criterion using the strain energy density factor (S) was verified by using the 3-parameters Weibull distribution function, the procedure of reliability verification is illustrated in Fig. 10. The values of Median rank, transformation function, and Weibull reliability were calculated from the following equations.

Median rank: $F=\frac{j-0.3}{N+0.4}\times 100\ (\%)$ (6)

Transformation function: $T=-\ln(1-F)$ (7)

Weibull reliability:

$$R(x)=\exp\left[-\left(\frac{x-c}{b}\right)^{a}\right] \quad (8)$$

a: shape parameter, b: scale parameter, c: location parameter, j: $j_{th}$ sample number, N: total number of sample, x: number of cycle to failure

From the the result, reliability of the fatigue strength assessment method using the strain energy density factor was estimated at about 85%. It means that fatigue design criterion for spot-welded lap joint of various dimensions can be predicted by this method.

## 6. Conclusions

In order to develop the method of fatigue strength assessment for spot-welded joints with various dimensions, considering the nugget edge of the spot weld as a ligament crack, the strain energy density factors were numerically analyzed. And then, fracture mechanical fatigue strength assessment was conducted by using the strain energy density factor synthetically including the three kinds of fracture modes. And also, its reliability and performance were verified by the theory of Weibull probability distribution.

From the analyis, the following conclusions are obtained:

(1) The strain energy density factor of IB type spot-welded joint was fracture mechanically calculated by considering around the spot weld as a ligament crack under the mixed Mode of Mode I, II and III. According to the increase of joint angle, $K_I$ was nearly constant, while $K_{II}$ and $K_{III}$ are changed. In particular, $K_{III}$ is linearly increased as the joint angle increased.

(2) Fatigue strength of the various IB type spot-welded joints subjected to tensile shear load could be more systematically rearranged by the strain energy density factor than by $\Delta P-N_f$ relation. And, the fatigue limit by the strain energy density factor was predicted in the range of $1.0\sim2.0\times 10^5 N/m$.

(3) Reliability of the fatigue strength assess-

ment method using the strain energy density factor was estimated at about 85%. Thus, it is expected that this method will be applicable to predict fatigue design criterion for spot-welded lap joint having various dimensions.

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